Premultiplying Eq. (21) by Eq. (18) will yield the initial value solution to Eqs. (1) and (4), which constitute the transition matrix as given by Table 1 of Ref. 1.

Discussion

Aside from its intrinsic usefulness for navigation-guidance mechanization, it was found that the locally level in-plane, out-of-plane frame of reference was the most convenient for performing the integration primarily because of the decoupling that takes place between in-plane and out-of-plane coordinates. The simplification which this decoupling provides eases immeasurably the analytic manipulations. Needless to say, these manipulations are still quite laborious whether one perturbs the integrals or integrates the perturbations.

The relationship of Eqs. (6–8) to Geyling's equations[4] should be noted. If Geyling had performed the transformation equation (5), then Eqs. (6–8) would result. Without this transformation, the integral would have been difficult to obtain.

Danby[3] also gives a detailed theory for two-body error coefficients. His matrizants are developed as perturbations of two-body integrals with respect to the orbit constants. The transition matrix is obtained upon multiplying two 6×6 matrizants, each evaluated at the respective epoch. It should be noted that the Q and $Q_0{}^{-1}$ matrices here obtained are, in reality, matrizants with respect to a different set of constants.

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Flow Field in a Swirl Chamber

D. H. Tsai*

Institut für Hochtemperaturforschung der Technischen Hochschule Stuttgart, Germany

Introduction

The operation of a swirl-stabilized plasmajet generator has been described recently by Pfender.[1] Figure 1 shows such a chamber with its typical dimensions. Gas is introduced into the chamber from symmetrically arranged peripheral jets at one (or both) end of the chamber and leaves from the central holes in the end plates. The tangential component of the jets drives the gas in rotation. As a plasmajet generator, the end plates serve as electrodes for discharging an arc along the chamber axis. The gas thus heated by the arc streams out of the central holes and forms plasmajets. The arc column inside the chamber is quite stable, but the exact mechanism of the stability is not clear.

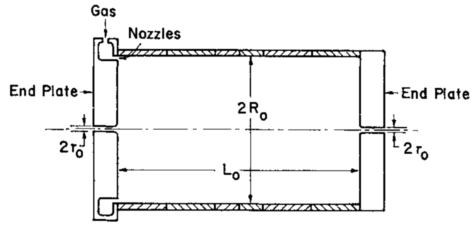

Fig. 1 Schematic diagram of a swirl chamber. In the experimental setup, $D_0 = 2R_0 = 60$ mm, $L_0 = 30$–100 mm, $2r_0 = 2$mm. Six nozzles each with exit area of 1 mm², nozzle axes at 10° from swirl chamber end plate.

The purpose of this note is to describe some quantitatively consistent measurements on the flow field in a series of swirl chambers with different L_0/D_0 ratios. These measurements were made without the arc along the axis. Other experiments, however, have shown that the radial pressure distribution remained qualitatively unaltered in the presence of the arc. A fuller account of this work and a discussion of the influence of the mass flow on the arc column stability will be published elsewhere.

Experiments

The quantities measured were the tangential component of the flow velocity, the flow direction, and the total pressure at different radial and axial positions in the swirl chamber. The tangential velocity components were measured by means of a series of flat-sided propellers (anemometers) mounted on jewel bearings located along the chamber axis. By assuming the gas in the central core to rotate as a solid body and the drag coefficient of the propeller element to remain constant, it was possible to evaluate the rotational speed ω of the gas core from the change of the rotational speed of the propeller ω_p due to a small increase in the propeller diameter. From this information, and with similar assumptions, it was possible to obtain the rotational speed of successively larger cylindrical gas layers as the propeller diameter was successively increased.

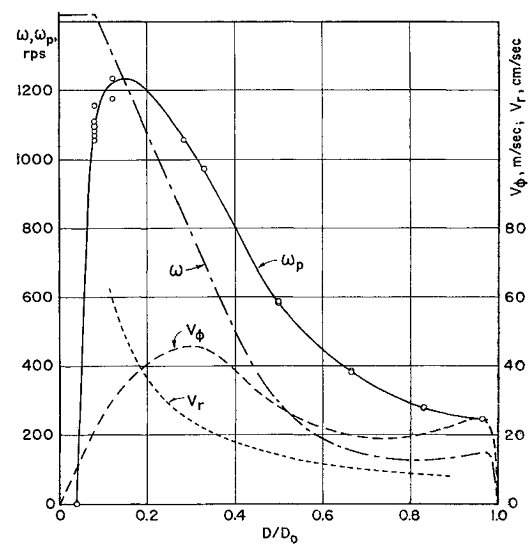

Fig. 2 Typical variation of the measured propeller rotational speed ω_p with propellers of different diameters D. For this series, the propellers were located at $L/L_0 = 0.45$ from the nozzle end, $L_0 = 100$ mm. Upstream nozzle pressure $P_p = 1.0$ kg/cm.² The rotational speed ω and the tangential velocity component V_ϕ of the gas were obtained from the ω_p curve. The radial velocity component V_r was computed from the measured flow rate.

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* Now Aeronautical Research Engineer, National Bureau of Standards, Washington, D. C.

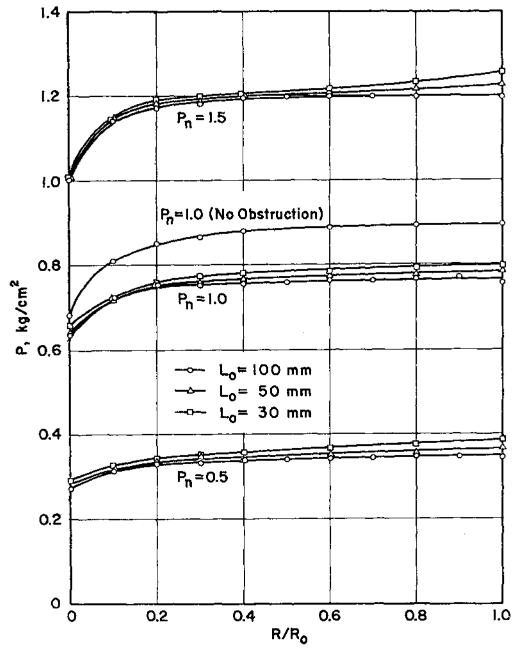

Fig. 3 Radial variation of total pressure P at $L/L_0 = 0.43$. To make these curves comparable to those in Fig. 2, the propeller support was placed in the chamber in order to produce comparable obstruction to flow. The curve marked "no obstruction" shows that the pressure distribution remained similar without obstruction.

The flow direction was measured by means of a hinged flag. By orienting the hinge axis in two mutually perpendicular directions (e.g., parallel and perpendicular to the chamber axis) and observing the flag positions, it was possible to determine the flow direction as the intersection of the two flag planes, except for some corrections that must be applied to account for the effect of radial pressure gradients in the flow field. At the walls, the flow directions were determined from the streaking marks.

The total pressure was measured by means of a pitot tube in the conventional manner.

The flow data obtained this way were checked against the mass rate of flow, which was also measured. The agreement was satisfactory.

Results and Discussion

Figures 2 and 3 show typical results of the rotational speed and total pressure measurements, respectively. In Fig. 2, the qualitative agreement of the V_ϕ vs D/D_0 curve with similar curves computed for the vortex tube, by Suzuki,[2] for example, is evident. The main result, however, was that the flow profile elsewhere in the chamber (except in the boundary layers near the solid walls) remained essentially the same as in these figures. This result was not altered by shortening the chamber length by a factor of three or by stopping one of the central holes in the end plate. Thus, in the range of variables investigated here, the flow field in the interior of the chamber had the characteristics of a strong two-dimensional vortex superimposed on a weak sink at the center. No important secondary flow[3] was observed. Of special interest was the falling pressure toward the chamber axis (Fig. 3) and the accompanying radial flow inward along the entire axis. It is conjectured that these features have a strong stabilizing influence on the arc column along the axis when such a chamber is used as a plasmajet generator.

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Pressure Variation in a Tank Undergoing an Acceleration

R. C. Progelhof*
Lehigh University, Bethlehem, Pa.

Nomenclature

a = speed of sound
L = length
p = pressure
t = time
u = magnitude of flow velocity
x = distance
α = acceleration
γ = ratio of specific heats
τ = nondimensional time

Subscript

0 = initial conditions

Introduction

In a recent technical note by Ehlers,[1] a solution was presented for the linearized equations of motion for the one-dimensional homentropic flow of a perfect gas in a cylindrical vessel of uniform cross-sectional area undergoing a unit constant acceleration in the direction of the major axis of the vessel. However, it is not necessary to linearize the equations of motion in order to obtain the pressure distribution in the vessel as a function of time, but a numerical or graphical technique based on the method of characteristics can be readily used. This note outlines the graphical technique that was used to obtain the pressure variation in the vessel as a function of time for two acceleration rates that are constant with time. From the pressure-time results obtained from the method of characteristics, an estimate can be made as to the maximum nondimensional acceleration for which the linearized approach can be used to approximate the pressure distribution in the vessel.

Theory

The four quasilinear equations of motion are the continuity equation (1), momentum equation (2), and Eqs. (3) and (4), relating the independent variables x and t and dependent variables a and u with four unknown partial derivatives[3,4]:

$$a\frac{\partial u}{\partial x} + \frac{2}{\gamma - 1}\frac{\partial a}{\partial t} + \frac{2u}{\gamma - 1}\frac{\partial a}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{2a}{\gamma - 1}\frac{\partial a}{\partial x} = \alpha(t) \tag{2}$$

$$\frac{\partial u}{\partial t}dt + \frac{\partial u}{\partial x}dx = du \tag{3}$$

$$\frac{\partial a}{\partial t}dt + \frac{\partial a}{\partial x}dx = da \tag{4}$$

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* Assistant Professor, Department of Mechanical Engineering. Member AIAA.